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Crack propagation and compliance calibration in fibre-reinforced polymers

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The critical stress intensity factor is used as a criterion for fracture since its use facilitates specification of critical defect size or critical fracture stress. It is shown experimentally that the compliance/crack length relationship for a fibre-reinforced material is significantly different to that of an elastic continuum. The difference is attributed to the different content of elastic strain energy in the composite, which is, in fact, elastically anisotropic and inhomogeneous. The experimental methods used lead towards practical design against fracture in composites.

1. Introduction

The first mode of failure to be designed against in fibre-reinforced polymers is likely to be brittle fracture. Design should, therefore, be based on a fracture theory which stands up to experimental verification. Theoretical studies indicate that the toughness, or fracture resistance of an orthotropic body depends on a number of elastic constants [1]. Economy of design is aided if a minimum number of material properties are required to be measured in order to characterize the fracture resistance of a material manufactured by a particular production process. The objective proposed here is to show how the fracture resistance and critical defect size of a material may be accurately specified with a minimum amount of testing.

The methods of linear elastic fracture mechanics (LEFM) have been applied to reinforced polymers by a number of authors [2–5]. Often the elastic analysis which accounts for the finite size and shape of an isotropic elastic body is directly applied. In these cases the critical stress intensity factor, K_C, concept is shown to be an adequate criterion for the onset of fracture even though it is clear that K_C does not describe the same stress field singularity as that in an isotropic elastic solid [6].

The LEFM concepts characterize the fracture resistance of a material in a number of equivalent ways. The K_C corresponds to a critical level of local stress at the crack tip; the critical energy release rate, G_C, specifies energy of crack extension; the critical plastic zone size specifies the range of preparatory plastic damage necessary before crack extension can commence. Tirosh [6] has shown that, for cracks extending at 90° to the fibres in a unidirectional composite, the preparatory plastic zone extends at 90° to the plane of crack extension, but has a length proportional to $\sigma^2 a$ where σ is the gross tension stress at infinity and a is the notch or crack length. This is common to plastic zones reaching ahead to the crack on the cracking plane, in isotropic materials, though the constant of proportionality found by Tirosh involves five independent material properties.

A number of workers have applied the G_C criterion of fracture [2, 3] in fibre composites and found that it is an adequate criterion for fracture and in some cases may be related to a K_C by the use of four independent material properties [2]. The use of a K_C criterion is valuable since the K_C value may be directly evaluated from a fracture load if an appropriate K-calibration curve is available, and will enable critical defect sizes to be specified for given working loads.

K-calibration curves may be produced by stress analysis methods, and many exist now for various shapes of elastically isotropic solids [7]. The K-calibration curve is essentially related to the elastic strain energy content of the body which

may be measured by compliance methods [8]. It would, therefore, seem fortuitous to us if the K-calibration curve derived by stress analysis for an isotropic and homogeneous elastic body could be used to derive a true K_C value (that is a K_C which may be related to a true G_C) for a fibre composite, although indeed Guest and Hoover [2] found that this was so for a particular material. In contrast, Ellis and Harris [3] find that G_C values obtained from K_C values derived on the basis of the K-calibration curve for an isotropic elastic continuum, do not agree with G_C values arrived at by calibrating with experimental compliance measurements.

It is our intention here to show how a true K_C value may be established for a fibre composite in such a way as to lead to specification of critical defect sizes. In a second paper we shall show how this method may be extended to account for variations of the fibre direction and the crack angle with respect to a tensile stress axis.

2. Experimental methods and materials

The material used was a commercial glass fibre/epoxide resin composite, Permaglass XE5, supplied by Permali Ltd. This composite is normally surfaced with a glass weave layer, but this surfacing was omitted from the material used for these experiments and this was, therefore, a simple unidirectional reinforced sheet 3.1 mm thick. The sheet had a specific gravity of 2.10, a tensile strength of 1295 MN m^{-2} in the direction of the fibres, and the fibre volume fraction was 0.66.

Specimens were cut from the 3.1 mm thick sheet using a diamond saw. Single edge notch tension pieces [7] were cut with an overall length of 190 mm, and holes for pin grips 140 mm apart. These holes were 8 mm in diameter and soft aluminium sheet was glued to the side surfaces surrounding the holes in order to facilitate gripping with screwed clamps passing through the holes. Universal joints in the loading bars ensured axial stressing. The single side notch was cut again with a diamond saw to ensure good surfaces, and was 2 mm wide. For compliance measurements the notch depth was varied, although for toughness measurements the notch depth gave a ratio of notch depth, a, to a gross width, w, of 0.5. For toughness tests the 2 mm wide notch was sharpened by being extended 1.5 mm using a jewellers saw. The final notch was 0.25 mm wide.

Compliance measurements on these specimens were carried out using an Instron load cell to measure the load and a dial gauge extensometer for displacements. These displacements were measured over a central guage length of 71 mm. The specimen surface was protected from damage from extensometer gripping points by gluing small discs of soft aluminium at the gripping points. Testing was carried out on an Instron machine at a cross-head speed of 0.2 mm min^{-1}.

In addition to compliance and toughness tests measurements of Young's modulus in the fibre direction and the Poisson's ratio corresponding to a contraction in the width direction was made on unnotched coupons. These measurements were made using longitudinally and transversely mounted strain gauges glued to the specimen with an epoxy resin.

3. Results

Compliance measurements were made for five values of a/w as shown in Fig. 1. A fourth degree

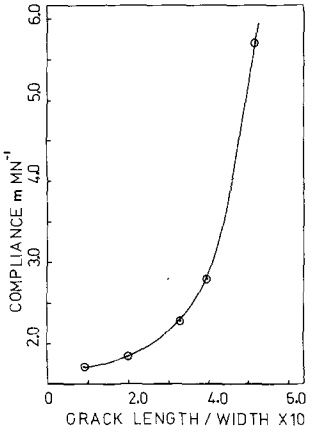

Figure 1 Compliance of notched specimens, expressed in m MN^{-1}, as a function of notch depth/gross width ratio.

polynomial was fitted to these results using an ICL 1905E computer. This equation was differentiated to obtain $dC/d(a/w)$, this quantity being numerically evaluated from $a/w = 0.1$ to 0.6, in steps of 0.05. Using the result of Irwin and Kies [8], namely:

$$G = (P^2/2B)(dC/da) \quad (1)$$

where B is the sample thickness, and the plane strain relation:

$$K^2 = EG/(1-\nu^2), \quad (2)$$

we obtain:

$$K^2 = \left(\frac{E}{1-\nu^2}\right)\left(\frac{P^2}{2B}\right)\left(\frac{dC}{da}\right). \quad (3)$$

Comparison with

$$K = \frac{YPa^{1/2}}{BW} \quad (4)$$

results in:

$$Y = \left(\frac{EB}{2(1-\nu^2)(a/w)}\right)\left(\frac{dC}{d(a/w)}\right)^{1/2} = \frac{KBW}{Pa^{1/2}} \quad (5)$$

The value of Y plotted against a/w constitutes a K-calibration curve and, therefore, we plot Y obtained from compliance data versus a/w in Fig. 2. Fitting these results to a fourth degree polynomial results in:

$$Y = 3.69 + 6.12(a/w) - 118.8(a/w)^2 + 420.67(a/w)^3 - 350.26(a/w)^4 \quad (6)$$

This should be compared with the polynomial found by stress analysis [7] and by compliance tests [9] for elastically isotropic materials, namely:

$$Y = 1.99 - 0.41(a/w) + 18.70(a/w)^2 - 38.48(a/w)^3 + 53.85(a/w)^4. \quad (7)$$

This polynomial is also plotted in Fig. 2.

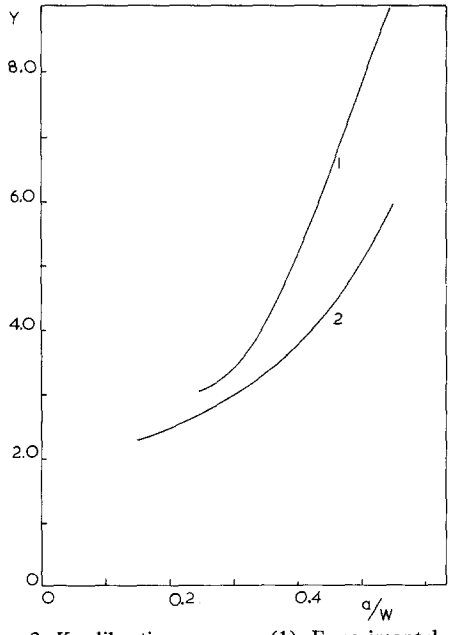

Figure 2 K-calibration curves. (1) Experimental compliance calibration from work herein. (2) Boundary collocation K-calibration from [7].

The value of Young's modulus measured on unnotched specimens was found to be consistently 4.72×10^4 MN m^{-2}; however, the value of Poisson's ratio varied significantly with load as shown in Fig. 3.

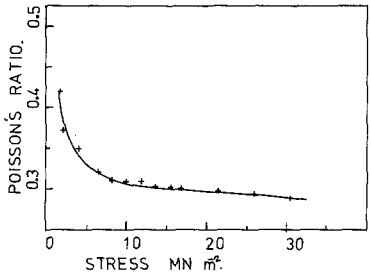

Figure 3 Variation of Poisson's ratio with tensile stress for unnotched specimens.

Fracture toughness testing was carried out using the methods recommended for metallic materials [10], with the exception that the single edge notched tension specimen described above was used rather than a bend test piece. A clip gauge [10] was mounted over the mouth of the notch and the load/crack opening displacement, (COD), recorded on an X–Y recorder. Fig. 4 shows a typical trace which is linear until failure commences. Progressive failure occurs on a rising load/COD curve. In order to define a fracture load the same rather arbitrary method as is used for metals was employed. That is failure is assumed to have occurred when the compliance has increased by 5% of its original value. This corresponds to locating the fracture load P_Q at the intersection of the curve of Fig. 4 with the secant of 5% less slope than the original elastic slope.

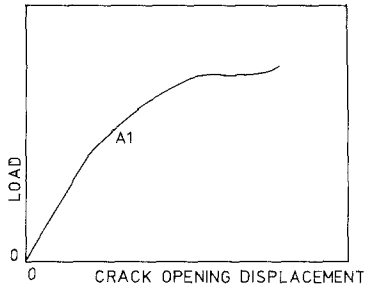

Figure 4 Trace of load versus crack opening displacement record for a notched specimen.

The fracture toughness K_C was evaluated from:

$$K_C = \frac{YP_Q}{BW} a^{1/2}. \quad (8)$$

The Y value was taken from tabled figures

Figure 5 Scanning electron micrograph of the crack surface at the notch tip showing delamination between the matrix and fibres, × 860.

corresponding to the compliance K-calibration curve of Fig. 2. The toughness measured was $14.4 \pm 0.8\,\mathrm{MN\,m^{-3/2}}$ which is close to the values obtained by Owen and Rose [11].

Failure initiated at the notch root in the form of intense plastic deformation spreading at 90° to the notch plane and along the direction of fibre reinforcement. Initiation of fracture was by delamination at the fibre/matrix interface or tensile fracture in the polymer close to this interface. Photomicrographs were taken of this fracture surface using a scanning electron microscope on samples coated with a gold cadmium layer in order to make the samples electrical conductors. Fig. 5 demonstrates the delamination between fibres and matrix.

4. Discussion

The K values used here are defined by the compliance change with crack extension and we may express the K definition through the finite width correction factor Y:

$$Y = \left[\frac{EB}{2(1-\nu^2)(a/w)} \cdot \frac{\mathrm{d}c \cdot}{\mathrm{d}(a/w)}\right]^{1/2} = \frac{KBW}{Pa^{1/2}} \qquad (9)$$

In this work the E and ν refer to extension along the direction of fibres and contraction in the transverse direction. Comparison of this Y value with that appropriate for homogeneously elastic materials shows that the latter is clearly not appropriate and is, therefore, not directly related to the elastic strain energy content of the body, which is indeed the driving force for fracture. The form of the expression for Y, above, is derived on the basis of elastic continuum behaviour, but is subsequently used as a definition for K. This means that the K used for the orthotropic material is defined in such a way as to make its form analogous with the K for a material behaving as an elastic continuum though its value is based on compliance measurements. The K for the orthotropic material does not define the same kind of stress singularity at the crack tip as we would have in the continuum [6], although it is related to a G, even though we have not used the detailed expression [1]:

$$G = \left[\frac{a_{11}a_{22}}{2}\right]^{1/2}\left[\left(\frac{a_{22}}{a_{11}}\right)^{1/2} + \frac{2a_{12} + a_{66}}{2a_{11}}\right]^{1/2} \cdot K^2 \qquad (10)$$

where the a_{ij} are the elements of the compliance tensor.

Indeed from a technological point of view our K value gives uniformity of treatment of composite and metallic materials, it is related to the G value and it allows identification of a critical load for a given defect size, a, or vice versa. This K is, therefore, an adequate criterion of fracture in that when $K = K_C$ failure occurs from the notch or crack even though that failure may arise because the stored strain energy does work in delamination at 90° to the notch plane rather than extending the notch in its own plane.

In terms of characterizing the toughness of the material examined here, the practical tests required are five compliance measurements and about five tests to fracture. From the compliance measurements the K-calibration curve for the material is established. The subsequent fracture tests establish the fracture load accurately. Thus a total of five specimens can yield all the required information.

In deriving the Y values from the compliance measurements the value of ν is chosen to be that appropriate for the fracture loads. In our case this was taken to be an average value for gross stress over $1000\,\mathrm{MN\,m^{-2}}$. Fig. 3 shows that ν is, in fact, relatively constant in this range.

The experimentation necessary to establish

fracture toughness of the unidirectional fibre-reinforced composite described here is not extensive and rules out any doubts or inaccuracies which arise from assuming continuum concepts for a Kcalibration. We shall show in a further paper that the unidirectional K-calibration may be used with modifications in cases where the notch or fibres, or both, are at an angle to the tensile axis.

5. Conclusions

A quantity K_C, analogous to the stress intensity factor for isotropic elastic materials, may be used to characterize the fracture resistance of fibre-reinforced materials. The appropriate K-calibration curve for the composite material must be used, and not the K-calibration curve for an isotropic material. This arises because the elastic energy content, shown by compliance measurements, is different for the composite. Use of the Srawley/Brown K-calibration gives rise to an error of around 20% at $a/w = 0.3$; but at $a/w = 0.5$ the error is in excess of 50%. Unfortunately the error is non-conservative and, therefore, for design requirements the compliance calibration is valuable.

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